Correlations in $e^+e^- \rightarrow W^+W^-$ hadronic decays

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Abstract

A mathematical formalism for the analysis of correlations in multi-source events such as W^+W^- production in e^+e^- annihilations is presented. Various measures used in experimental searches for inter-W correlations are reviewed.

1 Introduction

The problem of possible inter-W dynamical correlations continues to attract much experimental attention at LEP-II [1]. The importance of these in a precision measurement of the W-mass [2–4] is by far the main reason for the flurry of present activities. However, this should not be the sole motivation for careful experimental work in this subfield. For example, the possible absence of Bose-Einstein correlations (BEC) between pions originating from different W's raises interesting general and basic questions regarding the coherent versus incoherent nature of particle emission. Also the question of possible color- or string-reconnection effects may be of considerable importance for our understanding of the vacuum properties of Quantumchromodynamics.

As repeatedly emphasized by the Lund group [5, 6], Bose-Einstein correlations of a *coherent* type, for which we suggest the name "String Symmetrization Correlations (SSC)", are present in any string model of hadron production. Moreover, the SSC depend essentially only on *local* properties of the string and should thus be independent of the environment in which the string is fragmenting. This basic property is not in contradiction with experimental results on Bose-Einstein correlations in e^+e^- annihilations and lepton-nucleon scattering.

However, in systems comprising several strings, a *second type* of Bose-Einstein correlations may exist if the different strings behave independently so that they act as incoherent sources of particle emission. This second-order intensity correlation effect, or HBT effect, since first discovered by Hanbury Brown and Twiss [7], is expected to reflect the "geometry" of the collision process and, in particular, to be sensitive to the size of the "freeze-out" volume where the hadrons are formed. This volume could have an extension of many fermi's and can thus be significantly larger than the typical "radii" of less than a fermi, commonly measured *e.g.* in BEC studies of e^+e^- annihilations. As a result, correlation functions of identical bosons may well show an additional enhancement at substantially smaller values of their momentum difference than for SSC correlations.

The reaction $e^+e^- \to \mathrm{W^+W^-} \to \bar{\mathrm{q}}_1\bar{\mathrm{q}}_2\mathrm{q}_3\mathrm{q}_4 \to$ hadrons is a prototypical example of a system of which the hadronisation proceeds via the fragmentation of two color-fields or strings. Barring possible reconnection effects at an early stage of the evolution of the system, these strings are thought to fragment independently. Thus, SSC within single W's and inter-W HBT correlations could coexist in this system, but manifest themselves in different ranges of *e.g.* the commonly used variable Q^2, the square of the difference of the four-momenta of the identical pions. No dedicated searches in this direction have been made so far. Moreover, if the HBT correlations are of much shorter range than naively expected, the influence on W-mass measurements is likely to be much weaker than present studies suggest.

Limited statistics as well as limited experimental sensitivity at very small Q may prevent the HBT-type of correlation effect to be clearly observed in $e^+e^- \to W^+W^- \to$

$4q$ events. However, as yet unexplored alternative reactions exist. Much larger statistics is available in *e.g.* $e^+e^- \to q\bar{q} + g \to$ three-jet events. For these, due to the emission of a hard gluon, two color-disconnected strings are stretched, one between the quark and the gluon, another between the gluon and the antiquark. Depending on the origin of the identical pions studied, SSC as well as HBT correlations should contribute[1] to the correlations among identical bosons.

In hadron-hadron, hadron-nucleus and nucleus-nucleus collisions, hadroproduction is also believed to result from the break-up of several to very many strings. Since the superposition of many independent particle "sources" weakens the measured strength of correlations of the SSC type, HBT correlations may well dominate the Bose-Einstein correlations measured in these processes. This could also explain the observed positive correlation between particle density (or multiplicity) and the measured BEC radii. Finally, due to the "inside-outside" character of hadron production, whereby low-momentum particles "freeze out" first (in any reference frame), one may expect a correlation between the width of the Bose-Einstein enhancement and particle momenta if the HBT correlations are the dominant effect.

Returning to correlations in the WW system, experimental study of such effects should start from a well-defined mathematical framework. Since, in general, inter-W dynamics introduces genuine correlations between the decay products of the W's, we generalize the formalism presented in an earlier paper on the subject [8] where observables relevant for the case of stochastic independence and fully overlapping decays of the W's were proposed. We consider the general case of stochastic dependence and the separation of the W hadronic decay products in momentum space. We concentrate on second-order (two-particle) inclusive densities and correlation functions but the results can be generalized to higher orders.

In the mathematical treatment of the problem, a general inter-W correlation function is introduced, representing an arbitrary stochastic correlation between hadrons from different W's which could arise from color reconnection effects, Bose-Einstein correlations or others. Although most of the examples treated in this paper relate to Bose-Einstein studies involving identical particles, the formalism is general and can be used, where needed with a suitable change of kinematic variables, in other contexts as well.

2   Multivariate distributions and moments

Before discussing the problem of correlations among particles originating from W's decaying into fully hadronic, so-called four-jet configurations, $e^+e^- \to q_1\bar{q}_2q_3\bar{q}_4 \to$ hadrons, we first consider a more general problem.

Assume that a system (*e.g.* an event) comprising in total n (observed) particles,

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[1]I thank B. Andersson and G. Gustafson for discussions of this point.

can be subdivided into S possibly stochastically correlated groups or "sources" Ω_i ($i = 1, \ldots, S$). We take the n particles to be of identical type and assume that the groups are mutually exclusive i.e. every particle can be assigned to one and one group only[2]: $\Omega_i \cap \Omega_j = 0$, $\forall i, j$ and $i \neq j$. No assumptions are made, however, about possible overlap in momentum space of particles from different groups.

Let n be the total number of particles counted in the union of the S groups,

$$n = \sum_{m=1}^{S} n_m \quad . \tag{1}$$

Consider further the multivariate multiplicity distribution $P_S(n_1, \ldots, n_S)$ giving the joint probability for the simultaneous occurrence of n_1 particles in class Ω_1, $\ldots$, n_S particles in class Ω_S. The probability distribution of n is then given by

$$P(n) = \sum_{n_1=0}^{n} \cdots \sum_{n_S=0}^{n} P_S(n_1, \ldots, n_S) \delta_{n, n_1 + \cdots + n_S} \quad . \tag{2}$$

We further define the single-variate factorial moment generating function of $P(n)$

$$G(z) = \sum_{n=0}^{\infty} (1 + z)^n P(n) \, . \tag{3}$$

By expanding $(1 + z)^n$, (3) can be rewritten as

$$G(z) = 1 + \sum_{q=1}^{\infty} \frac{z^q}{q!} \sum_{n=0}^{\infty} \frac{n!}{(n-q)!} P(n), \tag{4}$$

$$= 1 + \sum_{q=1}^{\infty} \frac{z^q}{q!} \left\langle \frac{n!}{(n-q)!} \right\rangle, \tag{5}$$

$$G(z) = 1 + \sum_{q=1}^{\infty} \frac{z^q}{q!} \tilde{F}_q; \tag{6}$$

where in the last line we introduce the symbol $\tilde{F}_q$ for the (unnormalized) factorial (or binomial) moment of the probability distribution $P(n)$. The brackets in Eq. (5) denote a statistical average.

$$\tilde{F}_q = \langle n(n-1) \ldots (n-q+1) \rangle,$$
$$= \int_\Delta \mathrm{d}y_1 \ldots \int_\Delta \mathrm{d}y_q \, \rho_q(y_1, \ldots, y_q). \tag{7}$$

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[2]Such a subdivision can be based on a "natural" partition of the n particles in S groups according to the underlying dynamics. It could also be based on an experimentally dictated partition of the phase space in S distinct (non-overlapping) regions e.g. as the result of jet clustering.

The last equation expresses that, for identical particles, the factorial moment $\tilde{F}_q$ is equal to the integral over q-dimensional phase space (here for simplicity of notation represented by the variables y_i) of the q-particle inclusive density $\rho_q(y_1, \ldots, y_q)$ over the same phase space volume Δ [9]. The q-particle inclusive density $\rho_q(y_1, \ldots, y_q)$ is defined as

$$\rho_q(y_1, \ldots, y_q) = \frac{1}{\sigma} \frac{d^q \sigma}{dy_1 \ldots dy_q}, \tag{8}$$

with σ the total cross section of the considered reaction. Experimentally, this quantity is approximated by

$$\rho_q(y_1, \ldots, y_q) = \frac{1}{N_{\text{evt}}} \frac{dN^{\text{q-tuples}}}{dy_1 \ldots dy_q}, \tag{9}$$

with $dN^{\text{q-tuples}}$ the number of q-tuples of particles, counted in a phase space domain $(y_1 + dy_1, \ldots, y_q + dy_q)$; N_{evt} is the number of events in the sample.

Factorial cumulants are formally defined as the coefficients of $z^q/q!$ in the Taylor expansion of the function $\log G(z)$:

$$\log G(z) = \langle n \rangle z + \sum_{q=2}^{\infty} \frac{z^q}{q!} \tilde{K}_q. \tag{10}$$

The (unnormalized) factorial cumulants of order q, $\tilde{K}_q$, also known as Mueller moments [10], are equal to the q-fold phase space integral of the q-particle inclusive (so-called "connected" or "genuine") factorial cumulant correlation function $C_q(y_1, \ldots, y_q)$

$$\tilde{K}_q = \int_{\Delta} dy_1 \ldots \int_{\Delta} dy_q \, C_q(y_1, \ldots, y_q). \tag{11}$$

The correlation functions $C_q(y_1, \ldots, y_q)$ are, as in the cluster expansion in statistical mechanics, defined via the sequence

$$
\begin{aligned}
\rho_1(1) &= C_1(1), \\
\rho_2(1,2) &= C_1(1)C_1(2) + C_2(1,2), \\
\rho_3(1,2,3) &= C_1(1)C_1(2)C_1(3) + C_1(1)C_2(2,3) + C_1(2)C_2(1,3) + \\
&\quad + C_1(3)C_2(1,2) + C_3(1,2,3), \\
&\text{etc.}
\end{aligned}
$$

$$(12)$$
$$(13)$$

These relations can be inverted yielding

$$
\begin{aligned}
C_2(1,2) &= \rho_2(1,2) - \rho_1(1)\rho_1(2) \ , \\
C_3(1,2,3) &= \rho_3(1,2,3) - \sum_{(3)} \rho_1(1)\rho_2(2,3) + 2\rho_1(1)\rho_1(2)\rho_1(3) \ , \\
C_4(1,2,3,4) &= \rho_4(1,2,3,4) - \sum_{(4)} \rho_1(1)\rho_3(1,2,3) - \sum_{(3)} \rho_2(1,2)\rho_2(3,4)
\end{aligned}
$$

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$$+ 2 \sum_{(6)} \rho_1(1)\rho_1(2)\rho_2(3,4) - 6\rho_1(1)\rho_1(2)\rho_1(3)\rho_1(4), \tag{14}$$

$$\text{etc.} \tag{15}$$

In the above relations we have abbreviated $\rho_q(y_1, \ldots, y_q)$ to $\rho_q(1, 2, \ldots, q)$ etc.; the summations indicate that all possible permutations have to be taken (the number under the summation sign indicates the number of terms).

The factorial moments $\tilde{F}_q$ and factorial cumulants $\tilde{K}_q$ are easily found if $G(z)$ is known

$$\tilde{F}_q = \left. \frac{d^q G(z)}{dz^q} \right|_{z=0}, \tag{16}$$

$$\tilde{K}_q = \left. \frac{d^q \log G(z)}{dz^q} \right|_{z=0}. \tag{17}$$

The counting distribution $P(n)$ is likewise determined by $G(z)$

$$P(n) = \frac{1}{n!} \left. \frac{d^n G(z)}{dz^n} \right|_{z=-1}. \tag{18}$$

Let us now introduce the multidimensional S-variate generating function

$$G_S(z_1, \ldots, z_S) = \sum_{n_1=0}^{\infty} \sum_{n_2=0}^{\infty} \cdots \sum_{n_S=0}^{\infty} (1+z_1)^{n_1} \cdots (1+z_S)^{n_S} P_S(n_1, \ldots, n_S), \tag{19}$$

from which the S-variate factorial moments are easily obtained by differentiation:

$$\tilde{F}_{q_1 \ldots q_S} = \left\langle n_1^{[q_1]} \ldots n_S^{[q_S]} \right\rangle = \left. \left(\frac{\partial}{\partial z_1}\right)^{q_1} \cdots \left(\frac{\partial}{\partial z_S}\right)^{q_S} G_S(z_1, \ldots, z_S) \right|_{z_1 = \cdots z_S = 0}. \tag{20}$$

Likewise, S-variate factorial cumulants are obtained by differentiation of $\log G_S(z_1, \ldots, z_S)$.

Returning to the function $G(z)$ (3), it is not difficult to see that it can be written in terms of the multivariate generating function (19) as

$$G(z) = G_S(z_1, \ldots, z_S)|_{z_1 = z_2 = \cdots = z_S = z}. \tag{21}$$

Equation (21) therefore allows to express the factorial moments of n in terms of the multivariate factorial moments of $\{n_1, \ldots, n_S\}$.

Application of the Leibnitz rule

$$\left(\frac{d}{dz}\right)^q f(z) = \sum_{\{a_j\}} \frac{q!}{a_1! a_2! \ldots a_S!} \left(\frac{d}{dz}\right)^{a_1} f_1(z) \cdots \left(\frac{d}{dz}\right)^{a_S} f_S(z)$$

to the function

$$f(z) = f_1(z) \cdots f_S(z)$$

and using (21) leads immediately to the relation

$$\tilde{F}_q = \sum_{\{a_j\}} \tilde{F}^{(S)}_{a_1 \ldots a_S} \frac{q!}{a_1! \ldots a_S!}. \tag{22}$$

The summation is over all sets $\{a_j\}$ of non-negative integers such that

$$\sum_{j=1}^{S} a_j = q.$$

Formula (22) is a generalization for factorial moments of the usual multinomial theorem.

Likewise, taking the natural logarithm of both sides of (21), one obtains an identical relation as (22) among single-variate and multivariate factorial cumulants.

As an example, for two groups ($S = 2$) one finds from (22)

$$\tilde{F}_2 = \tilde{F}^{(2)}_{02} + 2\tilde{F}^{(2)}_{11} + \tilde{F}^{(2)}_{20}, \tag{23}$$

$$\tilde{F}_3 = \tilde{F}^{(2)}_{03} + 3(\tilde{F}^{(2)}_{12} + \tilde{F}^{(2)}_{21}) + \tilde{F}^{(2)}_{30}, \tag{24}$$

$$\tilde{F}_4 = \tilde{F}^{(2)}_{04} + 6\tilde{F}^{(2)}_{22} + 4(\tilde{F}^{(2)}_{13} + \tilde{F}^{(2)}_{31}) + \tilde{F}^{(2)}_{40}. \tag{25}$$

The factorial moments $\tilde{F}_{0i}$, $\tilde{F}_{i0}$, are determined from the counting distribution in a single group. The univariate factorial moments $\tilde{F}_q$ are obtained from the sum of counts in the two groups. The "mixed" factorial moments $\tilde{F}^{(2)}_{ij}$ ($i, j \neq 0$) express inter-group stochastic dependences. The relations (23)-(25) are trivially extended to more than two groups.

3 W^+W^- correlations

3.1 Integral moments and cumulants

We now apply the general results from the previous section to the case of interest: the production of W^+W^- in e^+e^- annihilation, and their subsequent decay to four jets: $W^+W^- \to 4q \to$ hadrons. Here, the number of "sources" S is equal to two.

Eq. (23) is of particular interest. In a less formal way, it can be written as:

$$\tilde{F}_2 \equiv \langle n(n-1) \rangle = \langle n_1(n_1-1) \rangle + \langle n_2(n_2-1) \rangle + 2 \langle n_1 n_2 \rangle, \tag{26}$$

where n_1 and n_2 are the number of particles from the decay of W^+ and W^-, respectively. This equation could also be derived directly by noting that $n = n_1 + n_2$ and working out the expression for $\langle n(n-1) \rangle$. To derive relations for $S > 2$ or between factorial

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moments of higher order, it is evidently less cumbersome to make use of the generating functions and Eq.(22).

In absence of inter-W correlations *of whatever origin*, kinematical, dynamical, due to experimental selections and cuts, ..., one has

$$\langle n_1 n_2 \rangle = \langle n_1 \rangle \langle n_2 \rangle. \tag{27}$$

Exhibiting explicitly the presence of inter-W correlations we write (26) as

$$\tilde{F}_2 \equiv \langle n(n-1) \rangle = \langle n_1(n_1-1) \rangle + \langle n_2(n_2-1) \rangle + 2 \langle n_1 \rangle \langle n_2 \rangle \; (1+\delta_I), \tag{28}$$

with

$$\delta_I \equiv \langle n_1 n_2 \rangle / \langle n_1 \rangle \langle n_2 \rangle - 1 \neq 0, \tag{29}$$

a measure of positive or negative inter-W correlations. Similarly, the factorial cumulant $\tilde{K}_2$ can be written as

$$\tilde{K}_2 = \tilde{K}_{20} + \tilde{K}_{02} + 2 \langle n_1 \rangle \langle n_2 \rangle \; \delta_I. \tag{30}$$

This equation expresses the correlation function, integrated over full phase space, of the whole system in terms of the integrated correlation functions of each component separately, and of the integrated inter-W correlation.

3.1.1 Interlude

The quantity δ_I is related to the variances $D^2_{W^\pm}$ of the single-$W^\pm$, $W^\pm \to q\bar{q}$, and $W^+ W^- \to 4q$ multiplicity distributions via the relation

$$
\begin{aligned}
D^2_{WW} &= \left\langle (n - \langle n \rangle)^2 \right\rangle = \left\langle [(n_1 - \langle n_1 \rangle) + (n_2 - \langle n_2 \rangle)]^2 \right\rangle \tag{31} \\
&= D^2_{W^+} + D^2_{W^-} + 2 \langle n_1 n_2 \rangle - 2 \langle n_1 \rangle \langle n_2 \rangle \tag{32}
\end{aligned}
$$

giving

$$\delta_I = \frac{D^2_{WW} - D^2_{W^+} - D^2_{W^-}}{2 \langle n_1 \rangle \langle n_2 \rangle}. \tag{33}$$

3.2 Differential distributions

In section 3.1, general relations were obtained relating factorial moments and cumulants of the multiplicity distributions of a $W^+ W^-$ system to those of its W^+ and W^- components. Since q-th order factorial moments are integrals over phase space of q-particle inclusive densities, we now turn to relations among the fully differential particle densities and correlation functions of a $W^+ W^-$ system and those of its composing parts, considering explicitly possible statistical dependences. We restrict the discussion to second-order densities and correlations.

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For two stochastically independent systems, we derived in an earlier paper [8] the relations

$$C_2^{WW}(1,2) = C_2^{W^+}(1,2) + C_2^{W^-}(1,2), \tag{34}$$

$$\rho_2^{WW}(1,2) = \rho_2^{W^+}(1,2) + \rho_2^{W^-}(1,2) + \rho_1^{W^+}(1)\rho_1^{W^-}(2) + \rho_1^{W^+}(2)\rho_1^{W^-}(1), \tag{35}$$

and further

$$\rho_1^{WW}(1) = \rho_1^{W^+}(1) + \rho_1^{W^-}(1). \tag{36}$$

Here $C_2^{WW}(1,2)$ and $C_2^{W^\pm}(1,2)$ are the two-particle correlation functions for $W^+W^- \to 4q$ events and $W^\pm \to 2q$ events, respectively; $\rho_1^{WW}(1)$, $\rho_1^W(1)$ are the corresponding single-particle inclusive densities.

Inspection of (30) suggest to write a *general* expression for $C_2^{WW}(1,2)$ as

$$C_2^{WW}(1,2) = C_2^{W^+}(1,2) + C_2^{W^-}(1,2) + \delta_I(1,2)\left\{\rho_1^{W^+}(1)\,\rho_1^{W^-}(2) + \rho_1^{W^+}(2)\,\rho_1^{W^-}(1)\right\}. \tag{37}$$

The function $\delta_I(1,2)$ describes correlations among different W's.

For $\delta_I(1,2) = 0$ (independent W's), (37) expresses the additivity[3] of the factorial cumulant correlation functions, a necessary and sufficient condition for stochastic independence [8].

The factors $\rho_1^{W^+}(1)\,\rho_1^{W^-}(2)$, $\rho_1^{W^+}(2)\,\rho_1^{W^-}(1)$ are introduced for normalization and explicitly account for differences in the single-particle densities of the two W's, as is the case when the momentum spectra of the decay products from different W's are not identical i.e. do not fully overlap[4].

With the definitions

$$\rho_2^{WW}(1,2) = C_2^{WW}(1,2) + \rho_1^{WW}(1)\,\rho_1^{WW}(2) \tag{38}$$

$$\rho_2^W(1,2) = C_2^W(1,2) + \rho_1^W(1)\,\rho_1^W(2) \tag{39}$$

and using (36) we can write a general form for $\rho_2^{WW}(1,2)$

$$\rho_2^{WW}(1,2) = C_2^{W^+}(1,2) + C_2^{W^-}(1,2) + \left\{\rho_1^{W^+}(1)\,\rho_1^{W^-}(2) + \rho_1^{W^+}(2)\,\rho_1^{W^-}(1)\right\}\delta_I(1,2)$$
$$+ \left\{\rho_1^{W^+}(1) + \rho_1^{W^-}(1)\right\}\left\{\rho_1^{W^+}(2) + \rho_1^{W^-}(2)\right\}. \tag{40}$$

Note that, in general, and in fully differential form, the terms $\rho_1^{W^+}(1)\,\rho_1^{W^-}(2)$ and $\rho_1^{W^+}(2)\,\rho_1^{W^-}(1)$ are *not* equal. $\rho_2^{WW}(1,2)$, however, must be symmetric in its arguments

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[3]For independent "sources", additivity is valid for all orders of the cumulant correlation functions.

[4]In [8] it was implicitly assumed that $\rho_1^{W^+}(1)\,\rho_1^{W^-}(2) = \rho_1^{W^+}(2)\,\rho_1^{W^-}(1) \equiv \rho_1^W(1)\,\rho_1^W(2)$, where $\rho_1^W(1)$ is the single-particle inclusive density of one W and with the further assumption that $\rho_1^{W^+}(1) = \rho_1^{W^-}(1) \equiv \rho_1^W(1)$. Ignoring possible charge-dependence, these equations are valid if the W^+ and W^- hadronic decay products overlap completely in momentum space.

for identical particles. Equation (40), together with (39) takes the form

$$
\begin{aligned}
\rho_2^{\mathrm{WW}}(1,2) \;=\;& \rho_2^{\mathrm{W}^+}(1,2) + \rho_2^{\mathrm{W}^-}(1,2) \\
& + \; \{1 + \delta_I(1,2)\} \left\{ \rho_1^{\mathrm{W}^+}(1)\,\rho_1^{\mathrm{W}^-}(2) + \rho_1^{\mathrm{W}^+}(2)\,\rho_1^{\mathrm{W}^-}(1) \right\}
\end{aligned}
\tag{41}
$$

Defining the experimentally often studied *normalized* two-particle density

$$
R_2^{\mathrm{WW}}(1,2) = \frac{\rho_2^{\mathrm{WW}}(1,2)}{\rho_1^{\mathrm{WW}}(1)\,\rho_1^{\mathrm{WW}}(2)},
\tag{42}
$$

one finds

$$
R_2^{\mathrm{WW}}(1,2) = \frac{\rho_2^{\mathrm{W}^+}(1,2) + \rho_2^{\mathrm{W}^-}(1,2) + \left\{ \rho_1^{\mathrm{W}^+}(1)\,\rho_1^{\mathrm{W}^-}(2) + \rho_1^{\mathrm{W}^+}(2)\,\rho_1^{\mathrm{W}^-}(1) \right\}\{1 + \delta_I(1,2)\}}{\rho_1^{\mathrm{W}^+}(1)\,\rho_1^{\mathrm{W}^+}(2) + \rho_1^{\mathrm{W}^-}(1)\,\rho_1^{\mathrm{W}^-}(2) + \rho_1^{\mathrm{W}^+}(1)\,\rho_1^{\mathrm{W}^-}(2) + \rho_1^{\mathrm{W}^+}(2)\,\rho_1^{\mathrm{W}^-}(1)}.
\tag{43}
$$

3.3   Correlations in the variable Q

The previous sections dealt exclusively with fully differential quantities. In practice, these are impossible to measure and a projection on a lower-dimensional space is needed. We here consider, for illustration, the kinematical variable $Q^2 = -(p_1 - p_2)^2$, the negative square of the difference in 4-momenta of particles 1 and 2. We use the notation $\rho^{\mathrm{W}^+} \otimes \rho^{\mathrm{W}^-}(Q)$ for integrals of the type

$$
\int\int d^3p_1 d^3p_2\, \rho^{\mathrm{W}^+}(1)\, \rho^{\mathrm{W}^-}(2)\, \delta\left(Q^2 + (p_1 - p_2)^2 \right).
\tag{44}
$$

In practical applications, such integrals are calculated using an event and track-mixing technique.

We assume from now on that $\delta_I(1,2)$ in (37) is a function of Q only: $\delta_I(Q)$. This simplifies the calculations but may not be fully realistic. At least for Bose-Einstein studies, it is known that the correlation function of like-sign pairs is not isotropic in four-momentum space. To simplify further we also assume that

$$
\rho_1^{\mathrm{W}^+} \otimes \rho_1^{\mathrm{W}^+}(Q) = \rho_1^{\mathrm{W}^-} \otimes \rho_1^{\mathrm{W}^-}(Q) \equiv \rho_1^{\mathrm{W}} \otimes \rho_1^{\mathrm{W}}(Q).
$$

The expressions in the previous sections take the following form

$$
\begin{aligned}
\rho_2^{\mathrm{WW}}(Q) \;=\;& \rho_2^{\mathrm{W}^+}(Q) + \rho_2^{\mathrm{W}^-}(Q) + 2\left\{1 + \delta_I(Q)\right\} \rho_1^{\mathrm{W}^+} \otimes \rho_1^{\mathrm{W}^-}(Q), & (45) \\
C_2^{\mathrm{WW}}(Q) \;=\;& C_2^{\mathrm{W}^+}(Q) + C_2^{\mathrm{W}^-}(Q) + 2\delta_I(Q)\left\{ \rho_1^{\mathrm{W}^+} \otimes \rho_1^{\mathrm{W}^-}(Q) \right\}. & (46)
\end{aligned}
$$

Integrating these expressions over all Q, we have

$$
\delta_I = \frac{1}{\langle n_{W^+}\rangle\,\langle n_{W^-}\rangle} \int dQ\, \delta_I(Q) \left\{ \rho_1^{\mathrm{W}^+} \otimes \rho_1^{\mathrm{W}^-}(Q) \right\}.
\tag{47}
$$

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Further

$$R_2^{WW}(Q) = \frac{\rho_2^{W^+}(Q) + \rho_2^{W^-}(Q) + 2\left\{1 + \delta_I(Q)\right\}\left\{\rho_1^{W^+} \otimes \rho_1^{W^-}(Q)\right\}}{2\left\{\rho_1^{W} \otimes \rho_1^{W}(Q) + \rho_1^{W^+} \otimes \rho_1^{W^-}(Q)\right\}}. \tag{48}$$

Introducing the "overlap function"

$$g(Q) = \frac{\rho_1^{W^+} \otimes \rho_1^{W^-}(Q)}{\rho_1^{W} \otimes \rho_1^{W}(Q)}, \tag{49}$$

we can rewrite (45)-(46) as

$$\rho_2^{WW}(Q) = \rho_2^{W^+}(Q) + \rho_2^{W^-}(Q) + 2\left\{1 + \delta_I(Q)\right\}g(Q)\,\rho_1^{W} \otimes \rho_1^{W}(Q), \tag{50}$$

$$C_2^{WW}(Q) = C_2^{W^+}(Q) + C_2^{W^-}(Q) + 2\delta_I(Q)\,g(Q)\,\rho_1^{W} \otimes \rho_1^{W}(Q). \tag{51}$$

Likewise

$$R_2^{WW}(Q) = \frac{\rho_2^{W^+}(Q) + \rho_2^{W^-}(Q) + 2\left\{1 + \delta_I(Q)\right\}g(Q)\,\rho_1^{W} \otimes \rho_1^{W}(Q)}{2\rho_1^{W} \otimes \rho_1^{W}(Q)\left\{1 + g(Q)\right\}}, \tag{52}$$

$$= \frac{1}{2}\frac{R_2^{W^+}(Q)}{1 + g(Q)} + \frac{1}{2}\frac{R_2^{W^-}(Q)}{1 + g(Q)} + \left\{1 + \delta_I(Q)\right\}\frac{g(Q)}{1 + g(Q)}. \tag{53}$$

Here $R_2^{W}(Q)$ is the normalized two-particle density for a single W

$$R_2^{W}(Q) = \frac{\rho_2^{W}(Q)}{\rho_1^{W} \otimes \rho_1^{W}(Q)} = 1 + K_2^{W}(Q), \tag{54}$$

with $K_2^{W}(Q)$ the normalized two-particle cumulant density.

The meaning of the various terms in (50) is the following: a term such as $\rho_2^{W^+}(Q)$ "counts" the number of like-sign pairs within a single W^+. Integrated over all Q, it equals $\langle n_{W^+}(n_{W^+} - 1)\rangle$; the term $2\left\{1 + \delta_I(Q)\right\}g(Q)\rho_1^{W} \otimes \rho_1^{W}(Q)$, "counts" the number of pairs (i, j) with particle i belonging to W^+ and particle j belonging to W^-. Its integral over all Q is equal to $2\langle n_{W^+}n_{W^-}\rangle$; the term $2g(Q)\rho_1^{W} \otimes \rho_1^{W}(Q) \equiv \rho_1^{W^+} \otimes \rho_1^{W^-}(Q)$ "counts" the number of pairs (i, j) in uncorrelated W's. Its integral equals $2\langle n_{W^+}\rangle\langle n_{W^-}\rangle$. These relations can serve as a check of any experimental method used to calculate the so-called "mixing" terms $\rho_1^{W} \otimes \rho_1^{W}(Q)$, $\rho_1^{W^+} \otimes \rho_1^{W^-}(Q)$.

To simplify further the expressions, but without essential loss of generality, we shall from now on assume that

$$\rho_2^{W^+}(Q) = \rho_2^{W^-}(Q) \equiv \rho_2^{W}(Q). \tag{55}$$

We obtain from (53)

$$R_2^{WW}(Q) = \frac{R_2^{W}(Q)}{1 + g(Q)} + \left\{1 + \delta_I(Q)\right\}\frac{g(Q)}{1 + g(Q)} = 1 + \frac{K_2^{W}(Q) + g(Q)\delta_I(Q)}{1 + g(Q)}, \tag{56}$$

$$K_2^{WW}(Q) = \frac{K_2^{W}(Q)}{1 + g(Q)} + \delta_I(Q)\frac{g(Q)}{1 + g(Q)}. \tag{57}$$

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where $K_2^{WW}(Q) = R_2^{WW}(Q) - 1$, is the normalized cumulant density of the WW system.

Both $K_2^{WW}(Q)$ and $K_2^W(Q)$ are often parameterized with Gaussians. However, from Eqs. (56)-(57), it is seen that neither $R_2^{WW}(Q)$ nor $K_2^{WW}(Q)$ will, in general, have the same functional Q-dependence as $K_2^W(Q)$ unless $K_2^W(Q) \sim \delta_I(Q)$ and $g(Q)$ is constant.

Consider next the following limiting forms of the previous expressions.

- **Fully overlapping and uncorrelated W-decays: $\delta_I(Q) = 0$.**

  Here, all factors $\rho_1 \otimes \rho_1(Q)$ are equal so that $g(Q) = 1$. Equations (56-57) become

  $$R_2^{WW}(Q) = \frac{1}{2}\left\{1 + R_2^W(Q)\right\}, \tag{58}$$

  $$K_2^{WW}(Q) = \frac{1}{2}K_2^W(Q). \tag{59}$$

  For this special case, the cumulant correlation function for $W^+W^- \to 4q$ events is only half that of a single W, corresponding to $W^+W^- \to 2q$ events. This result was first derived in [8]. Its validity basically rests on the additivity of factorial cumulants for sums of independent random variables[5].

- **Decreasing overlap, arbitrary $\delta_I(Q)$.** Above production threshold, and with increasing center-of-mass energy, $\sqrt{s}$, the decay products of the two W's will show a decreasing overlap in momentum space. The factor $\rho_1^{W^+} \otimes \rho_1^{W^-}(Q)$ which is a measure of this overlap, will tend to zero for any fixed value of Q when $s \to \infty$. For $g(Q) \to 0$ one obtains

  $$R_2^{WW}(Q) \rightarrow \frac{\rho_2^W(Q)}{\rho_1^W \otimes \rho_1^W(Q)} = R_2^W(Q), \tag{61}$$

  $$K_2^{WW}(Q) \rightarrow K_2^W(Q). \tag{62}$$

  This result is intuitively clear. The diminishing overlap of the W^+ and W^- decay products decreases the contribution from pairs of particles which originate from different W's. For any fixed Q, $R_2^{WW}(Q)$ increasingly receives contributions from pairs belonging to the same W only.

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[5]In general, for S *fully overlapping identical and independent* "sources" it follows from the additivity of cumulants that

$$\frac{\tilde{K}_q^{(S)}}{\langle n \rangle^q} = \frac{\tilde{K}_q^{(1)}}{S^{q-1}}, \tag{60}$$

with $\tilde{K}_q^{(S)}$ the unnormalized integrated factorial cumulant of a system composed of S sources, $\langle n \rangle$ its average multiplicity and $\tilde{K}_q^{(1)}$ the integrated factorial cumulant of a single source [11]. We note that, besides the assumptions mentioned, no further approximations are involved in deriving (60) in contrast to the derivation in [12] where (60) is only obtained under (unnecessary) further conditions.

Equations (56-57) show that, as a consequence of not fully overlapping "sources", genuine inter-source correlations, represented by $\delta_I(Q)$, are "diluted" in the actual measurement of $R_2^{\mathrm{WW}}(Q)$ or $K_2^{\mathrm{WW}}(Q)$ since $g(Q)/(1+g(Q))$ may become quite small in practical situations.

This has further implications. In experimental conditions where one is attempting to avoid the effect of inter-source correlations (such as in W-mass measurements), methods should be devised to render the overlap function as small as possible *e.g.* by suitably chosen cuts.

On the opposite, when the main interest is focussed on establishing inter-source correlations, one could try to devise methods that maximize $g(Q)/(1+g(Q))$ and thus increase the experimental sensitivity to $\delta_I(Q)$.

With respect to searches for inter-W correlations at LEP, it remains to be investigated how much the presently adopted cuts and algorithms used to select with high efficiency WW events, affect the shape of the overlap fuction and contribute to weaken (possible) genuine WW correlation effects in the analyzed data. The same holds for studies of possible color-reconnection effects.

With the formalism described above, it is now straightforward to obtain expressions for a variety of observables used in experimental studies of inter-W correlations. Some of these are reviewed in the following. Others are easily constructed using the results of this section.

4 Examples

4.1 The L3 test-statistics D and D'

The L3 collaboration, in a search for inter-W (BE) correlations, discussed in a recent paper [13] the distributions ("test-statistics")

$$\Delta\rho(Q) = \rho_2^{\mathrm{WW}}(Q) - 2\rho_2^{\mathrm{W}}(Q) - 2\rho_2^{\mathrm{mix}}(Q), \tag{63}$$

$$D(Q) = \frac{\rho_2^{\mathrm{WW}}(Q)}{2\rho_2^{\mathrm{W}}(Q) + 2\rho_2^{\mathrm{mix}}(Q)}, \tag{64}$$

$$D'(Q) = \frac{D(Q)}{D_{\mathrm{MC,\ no\ BEC}}(Q)}; \tag{65}$$

where $\rho_2^{\mathrm{mix}}(Q)$ is identical to $\rho_1^{\mathrm{W}^+} \otimes \rho_1^{\mathrm{W}^-}(Q)$. The ratio $D'(Q)$ is obtained by dividing $D(Q)$ by the same derived from a Monte Carlo calculation *without* Bose-Einstein correlations. As very similar quantity was studied by ALEPH [14].

These distributions can be rewritten as

$$\Delta\rho(Q) = 2g(Q)\,\delta_I(Q)\,\rho_1^{\mathrm{W}} \otimes \rho_1^{\mathrm{W}}(Q), \tag{66}$$

$$D(Q) \;=\; 1 + \delta_I(Q)\frac{g(Q)}{R_2^{\mathrm{W}}(Q) + g(Q)}, \tag{67}$$

$$D'(Q) \;=\; 1 + \delta_I(Q)\frac{g(Q)}{R_2^{\mathrm{W}}(Q) + g(Q)}. \tag{68}$$

Observe that (67) and (68) are formally, but not necessarily experimentally, identical[6].

These expressions show that none of the studied observables isolates completely the genuine inter-W correlation function $\delta_I(Q)$. It can be measured most directly via the unnormalized function $\Delta\rho(Q)$, as seen from (66), once the mixing term $\rho_1^{\mathrm{W}^+} \otimes \rho_1^{\mathrm{W}^-}(Q)$ is determined. This equation therefore (strongly) suggests to study the ratio

$$\frac{\Delta\rho(Q)}{2\rho_2^{\mathrm{mix}}(Q)} = \delta_I(Q). \tag{69}$$

It can be determined directly from data only.

4.2 The fraction of pairs from different W's

This distribution was used by L3 [15] and DELPHI [16]. The fraction of like sign pairs from *different W's* is, in our notation, defined as

$$F(Q) = \frac{2\left\{1 + \delta_I(Q)\right\} \rho_1^{\mathrm{W}^+} \otimes \rho_1^{\mathrm{W}^-}(Q)}{\rho_2^{\mathrm{WW}}(Q)}, \tag{70}$$

or

$$F(Q) = \frac{g(Q)\left\{1 + \delta_I(Q)\right\}}{R_2^{\mathrm{W}}(Q) + g(Q)\left\{1 + \delta_I(Q)\right\}}. \tag{71}$$

The equation can be solved for $g(Q)$ with the result

$$g(Q) = R_2^{\mathrm{W}}(Q)\,\frac{F(Q)}{1 - F(Q)}\,\frac{1}{1 + \delta_I(Q)}. \tag{72}$$

Using $F(Q)$ presented in [15], we have plotted $g(Q)$ and $g(Q)/(1 + g(Q))$ in Fig. 1. Since $\delta_I(Q)$ is unknown, we assume, for illustration, the "Goldhaber"-form $\delta_I(Q) = \Lambda\exp\left(-r_I^2 Q^2\right)$ and $R_2^{\mathrm{W}} = 1 + \lambda\exp\left(-r^2 Q^2\right)$, with $r = 0.67$ fm and $\lambda = 0.7$ taken from [13]. We set $r_I = r$.

The function $g(Q)/(1 + g(Q))$ has a value between $0.14 - 0.24$ at $Q = 0$, depending on Λ, implying (*cfr.* Eq. (57)) that the effectively measured strength of possible inter-W correlation effects is smaller by the same factor.

With $g(Q)$ derived from the L3 results, we calculated the quantity $D(Q)$ as a function of Λ. The result is shown in Fig. 2. As is clear from (67), $D(Q)$ *is not* of the form $\sim (1 + \tilde{\Lambda}\exp\left(-k^2 Q^2\right))$, used in [13].

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[6]Eq.(68) remains unchanged when it is calculated from a Monte Carlo with only intra-W correlations.

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To test for inter-W correlations, a density function, $R_2^{\mathrm{WW}}(Q)_{\mathrm{BES}}$, is defined in [15] which is supposed to coincide with the data distribution $R_{2,L3}^{\mathrm{WW}}(Q)$ (defined below) if inter-W correlations are absent. This function is stated to describe the data on $R_{2,L3}^{\mathrm{WW}}(Q)$, thus adding extra support to the claimed absence of inter-W BEC.

The quantity $R_{2,L3}^{\mathrm{WW}}(Q)$ is defined as the ratio of the like-sign two-particle density in the data to the same obtained from a Monte Carlo model without Bose-Einstein correlations. Assuming that the model reproduces perfectly $R_2^{\mathrm{W}}(Q)$, this quantity is given by

$$R_{2,L3}^{\mathrm{WW}}(Q) = \frac{R_2^{\mathrm{W}} + g(Q)\left\{1 + \delta_I(Q)\right\}}{1 + g(Q)} = 1 + \frac{K_2^{\mathrm{W}} + g(Q)\delta_I(Q)}{1 + g(Q)}. \tag{73}$$

The "test" function $R_2^{\mathrm{WW}}(Q)_{\mathrm{BES}}$ is taken to be

$$R_2^{\mathrm{WW}}(Q)_{\mathrm{BES}} = 1 + \left\{1 - F(Q)\right\} K_2^{\mathrm{W}}(Q), \tag{74}$$

with $F(Q)$ given by (71) and setting $\delta_I(Q) = 0$. Equation (74) can then be written as

$$R_2^{\mathrm{WW}}(Q)_{\mathrm{BES}} = 1 + \frac{1 + K_2^{\mathrm{W}}(Q)}{1 + K_2^{\mathrm{W}}(Q) + g(Q)} K_2^{\mathrm{W}}(Q). \tag{75}$$

With $\delta_I(Q) = 0$ in (73) for no inter-W correlations, it is seen that this distribution and that of Eq. (75) are different, unless $K_2^{\mathrm{W}}(Q) = 0$ or $g(Q) = 0$. We conclude that (74) is not suitable to test for the "null hypothesis" $\delta_I(Q) = 0$. An analogous test-distribution, used by the DELPHI Collaboration for similar purposes (see below), has the same defect.

4.3   The DELPHI observables

In the DELPHI analysis of Bose-Einstein correlations in WW events [16], the following ratio is studied

$$R_{4q}^{\mathrm{data}} = \frac{\rho_2^{\mathrm{WW}}(Q)}{\rho_{2,\mathrm{MC, \ no \ BE}}^{\mathrm{WW}}(Q)}, \tag{76}$$

where the denominator is calculated from a Monte Carlo without Bose-Einstein effects. This can be rewritten using (50) as

$$R_{4q}^{\mathrm{data}}(Q) = \frac{R_2^{\mathrm{W}}(Q)}{1 + g(Q)} + \left\{1 + \delta_I(Q)\right\}\frac{g(Q)}{1 + g(Q)} = 1 + \frac{K_2^{\mathrm{W}}(Q) + g(Q)\delta_I(Q)}{1 + g(Q)}, \tag{77}$$

which coincides with the general expression (56). We have assumed that, in the Monte Carlo, identical particle pairs are uncorrelated, i.e. $\rho_2^{\mathrm{W}}(Q) = \rho_1^{\mathrm{W}} \otimes \rho_1^{\mathrm{W}}(Q)$.

The function (77) is plotted in Fig. 3, together with R_2^{W} (denoted by R_{2q} in [16]) measured in the DELPHI analysis and used here as input. From the above expression,

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and from the figure, it is clear that this distribution remains sensitive to inter-W correlations. However, it cannot, in general, be described by the same parameterization as used for $R_2^W(Q)$, even for $\delta_I(Q) = 0$. Fig.3 also shows, for illustration, the same function using a Gaussian parameterization of $\delta_I(Q)$ with a "radius-parameter" of 3 fm. The latter might be typical for the range of second-order interference (HBT) correlations associated with incoherent strings. In this case, the Q-region affected by BEC is restricted to values below ~ 100 MeV and will decrease even further for still larger "radii".

In Fig. 4 we show the intercept, $R_{4q}^{data}(Q)$ at $Q = 0$, and that of $R_2^W(Q)$, as a function of Λ. It is seen that the intercept for the $WW \to 4q$ channel remains below that of $R_2^W(Q)$ for $\Lambda < \lambda$. It is, of course, independent of the values of the parameters r and r_I.

4.3.1 $R_{4q}(Q)$(mixing)

DELPHI also studied a "test" distribution, $R_{4q}(Q)$(mixing), constructed from mixed independent ($WW \to 2q$) events and defined as

$$R_{4q}(Q)(\text{mixing}) = \frac{\left[\rho_2^W(Q) + \rho_2^{mix}(Q)\right]_{\text{data}}}{\left[\rho_2^W(Q) + \rho_2^{mix}(Q)\right]_{\text{MoCa, No Be}}} \tag{78}$$

with $\rho_2^{mix}(Q) = \rho_1^{W^+} \otimes \rho_1^{W^-}$. The above expression is equal to

$$R_{4q}(Q)(\text{mixing}) = 1 + \frac{K_2^W(Q)}{1 + g(Q)}. \tag{79}$$

This expression indeed coincides with (77) for $\delta_I(Q) = 0$ and can thus be used to test for inter-W correlations. It is plotted in Fig. 3 and depends here (weakly) on $\delta_I(Q)$ which enters the definition of $g(Q)$. We remark again that $R_{4q}(Q)$(mixing) cannot, in general, be described by the same parameterization as used for $R_2^W(Q)$.

Defining $\Delta\lambda$(mixing) as the difference in intercepts at $Q = 0$ of $R_{4q}(Q)$(mixing) and R_{4q}^{data} we find

$$\Delta\lambda(\text{mixing}) = \Lambda \frac{g(0)}{1 + g(0)}, \tag{80}$$

with $\Lambda = \delta_I(0)$.

4.3.2 $R_{4q}(Q)$(linear)

A further quantity studied by DELPHI is $R_{4q}(Q)$(linear), defined in Eq. (25) of ref. [16]. It is constructed to supposedly coincide with $R_2^{WW}(Q)$ if inter-W correlations are ab-

sent. With our notation it is given by

$$R_{4q}(Q)(\text{linear}) = R_2^W(Q) + \left\{1 - R_2^W(Q)\right\} \frac{g(Q)}{R_2^W(Q) + g(Q)}. \tag{81}$$

This function is plotted in Fig. 5. The distribution differs from R_{4q}^{data} even for $\delta_I(Q) = 0$ (or $\Lambda = 0$) and, therefore, has not the intended properties. Nevertheless, from Fig. 3 and Fig. 5 it can be seen that $R_{4q}(Q)(\text{linear})$ and $R_{4q}(Q)(\text{mixing})$ are numerically quite close. The weak dependence on Λ is again due to the apriori unknown dependence of $g(Q)$ on $\delta_I(Q)$.

5 Summary

In this paper we described a mathematical formalism which should allow a systematic study and improved understanding of particle correlations in a physical system which is composed of S possibly stochastically correlated parts. Most emphasis was put on two-particle inclusive densities and correlation functions for $S = 2$, but the formalism can be extended to arbitrary orders and arbitrary S.

Taking the reaction $e^+e^- \rightarrow W^+W^- \rightarrow q_1\bar{q}_2q_3\bar{q}_4 \rightarrow$ hadrons as a prime example, it was shown how to relate the measurement of correlations within one hadronically decaying W to those measured in the full $WW \rightarrow 4q$ system.

For the observables used in present studies, correlations in fully hadronic WW decay's can be expressed in terms of the correlation function of a single W, an inter-W correlation δ_I and a function, g, which quantifies the degree of overlap in momentum space of the decay products from different W's.

On several examples, we have shown that some of the presently used techniques to search for inter-W correlations are not optimal. In some cases, experimental quantities have been used which lack the correct mathematical properties and are based on intuition rather than on sound mathematics.

The determination of the overlap function allows to assess quantitatively the sensitivity of a particular distribution to inter-W correlations. Moreover, the influence of experimental and methodological cuts can be systematically investigated .

An estimate of $g(Q)$, presented here, indicates that the fully hadronic, four-jet decay channel of the reaction $e^+e^- \rightarrow W^+W^-$ may not be optimal to search for inter-W Bose-Einstein effects. The usual event selections, requiring four well-isolated jets in the final state, weaken the sensitivity to possible inter-W correlations. Better results may well be obtained in analyses using less stringent cuts, or which include *e.g.* three-jet events with suitably chosen topology so as to keep the QCD background at a tolerable level.

Our survey of some of the test-distributions used so far indicates that the observable $\Delta\rho(Q)$, defined in Eq. (63), proposed in [8] and measured by L3 [13], provides a direct

measure of possible inter-W correlations. It suggests itself as suitable observable well-adapted for a global LEP-wide combination of WW data.

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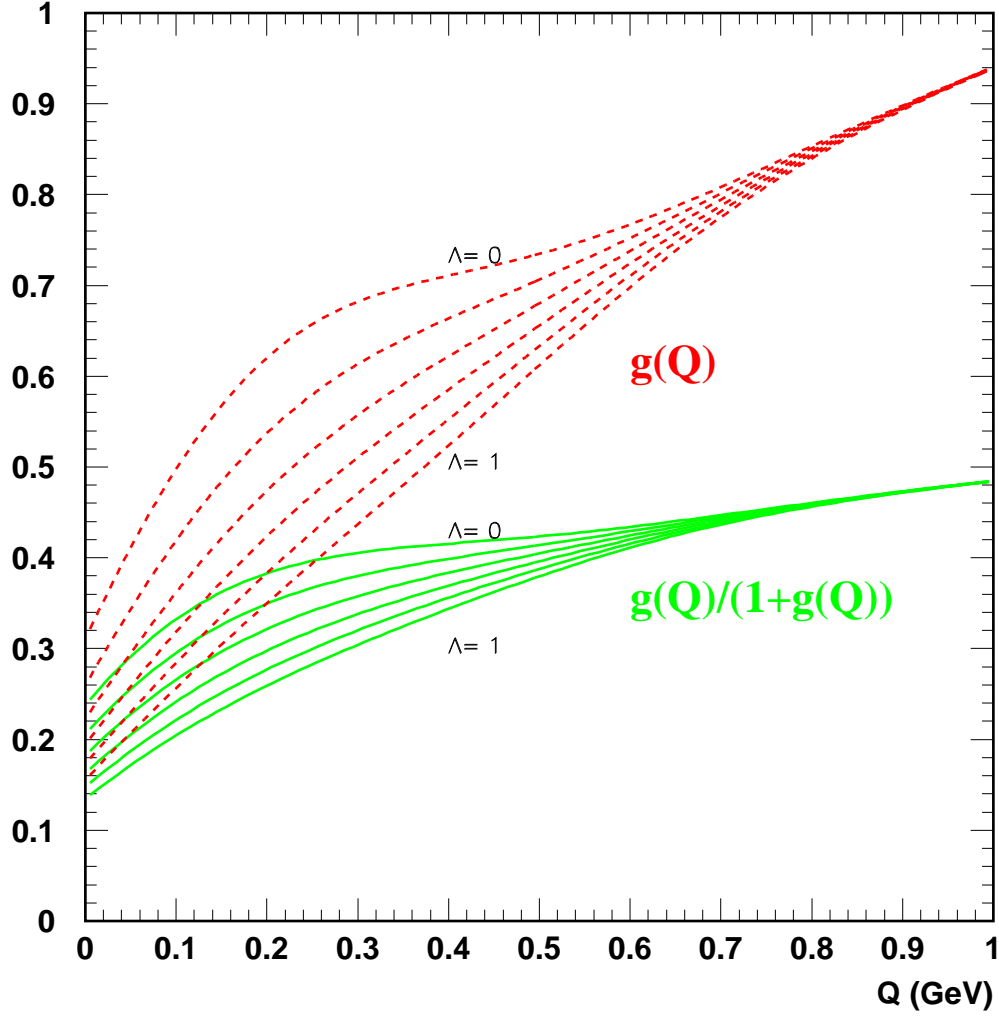
Figure 1: L3: the functions $g(Q)$ and $g(Q)/(1 + g(Q))$; $g(Q)$ is calculated assuming the form $\delta_I(Q) = \Lambda \exp\left(-r_I^2 Q^2\right)$, $r_I = 0.67$ fm, with Λ varying in the range $0.0 - 1.0$, in steps of 0.2.
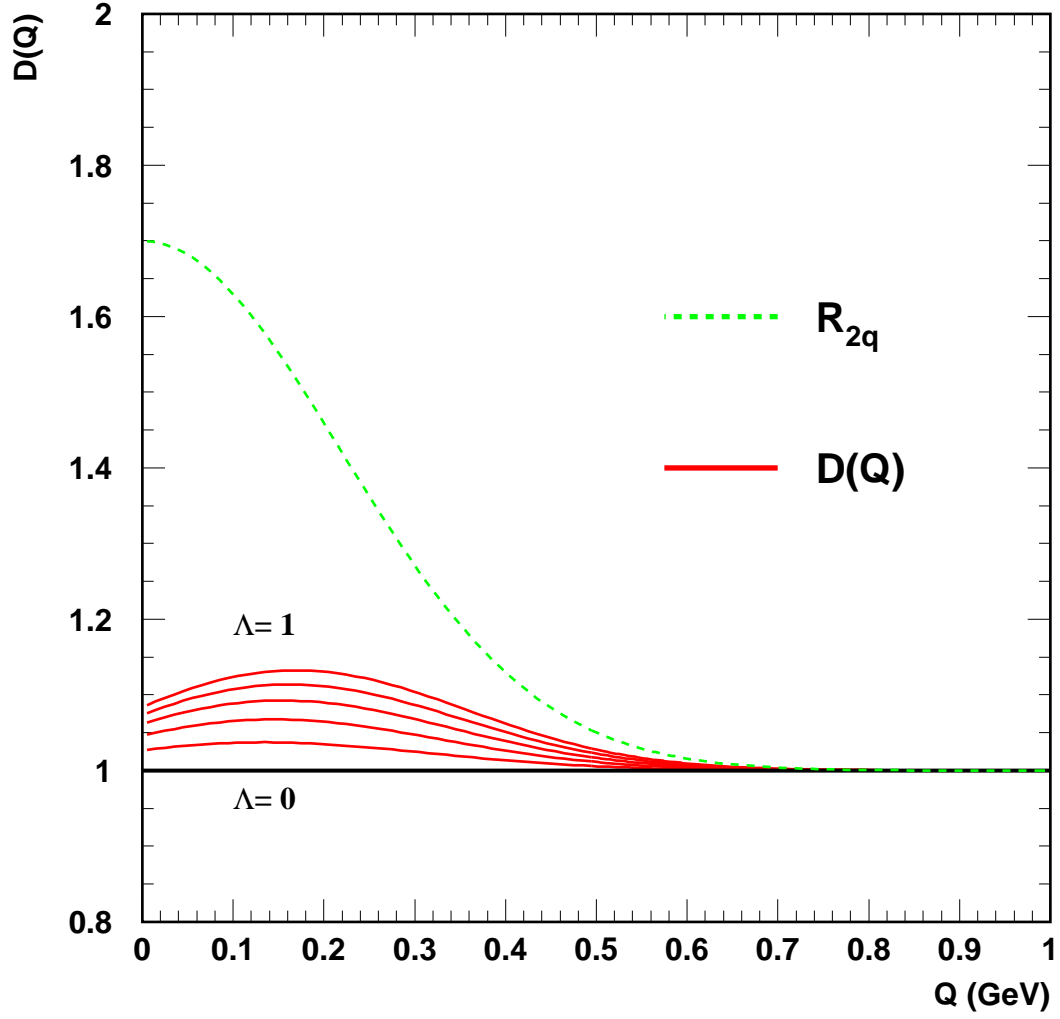
Figure 2: The test-statistic $D(Q)$ used by L3, Eq.(67). Also shown is the L3 parameterization of $R_{2q}(Q) \equiv R_2^W(Q)$.
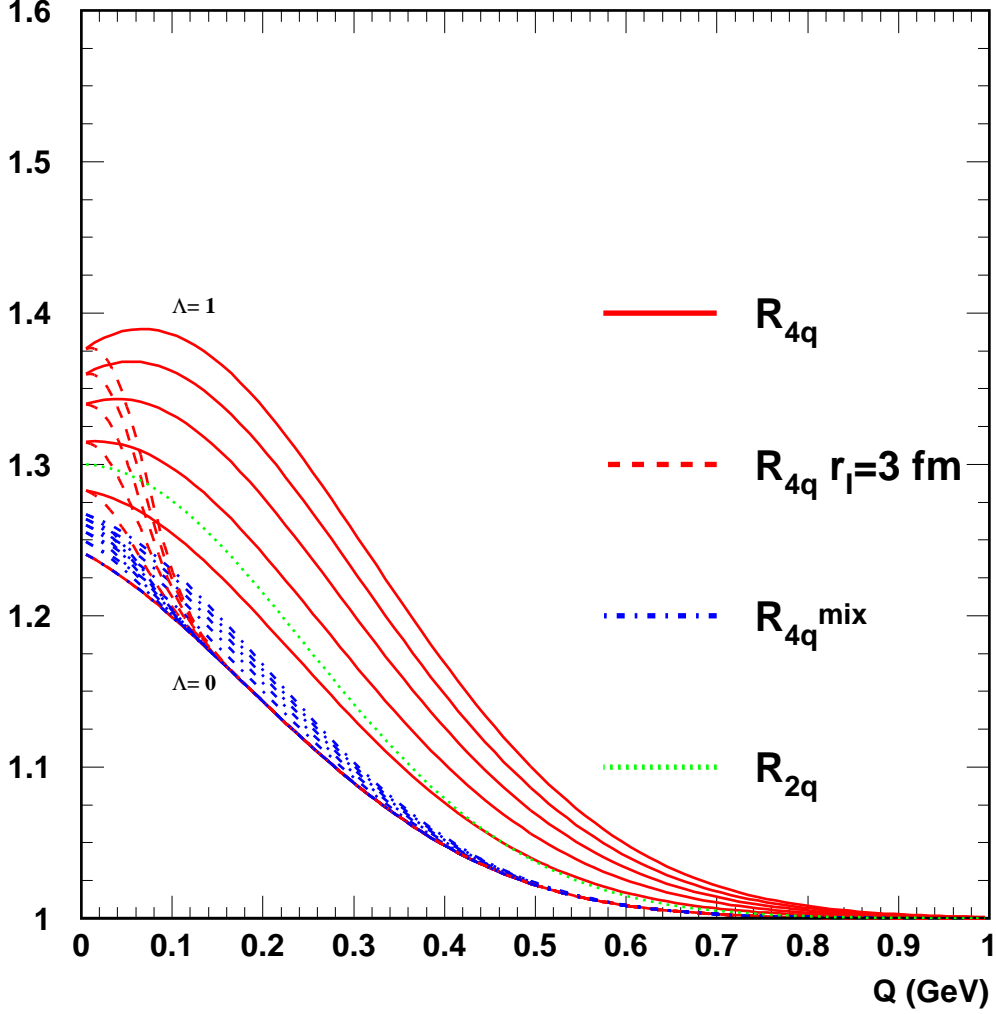
Figure 3: DELPHI: The normalized densities $R_{4q}(Q) = R_2^{\text{WW}}(Q)$, Eq.(77) in the text, using as input the DELPHI parameterization of $R_{2q}(Q) \equiv R_2^{\text{W}}(Q)$ and assuming $\delta_I(Q) = \Lambda \exp\left(-r^2 Q^2\right)$; Λ varies in the range $0.0 - 1.0$, in steps of 0.2 The dashed curves show the same with $r = 3$ fm. Also shown (dot-dashed) is the distribution, Eq.(79), corresponding to mixed independent $WW \to 2q$ events. It depends implicitly on $\delta_I(Q)$ through the function $g(Q)$.
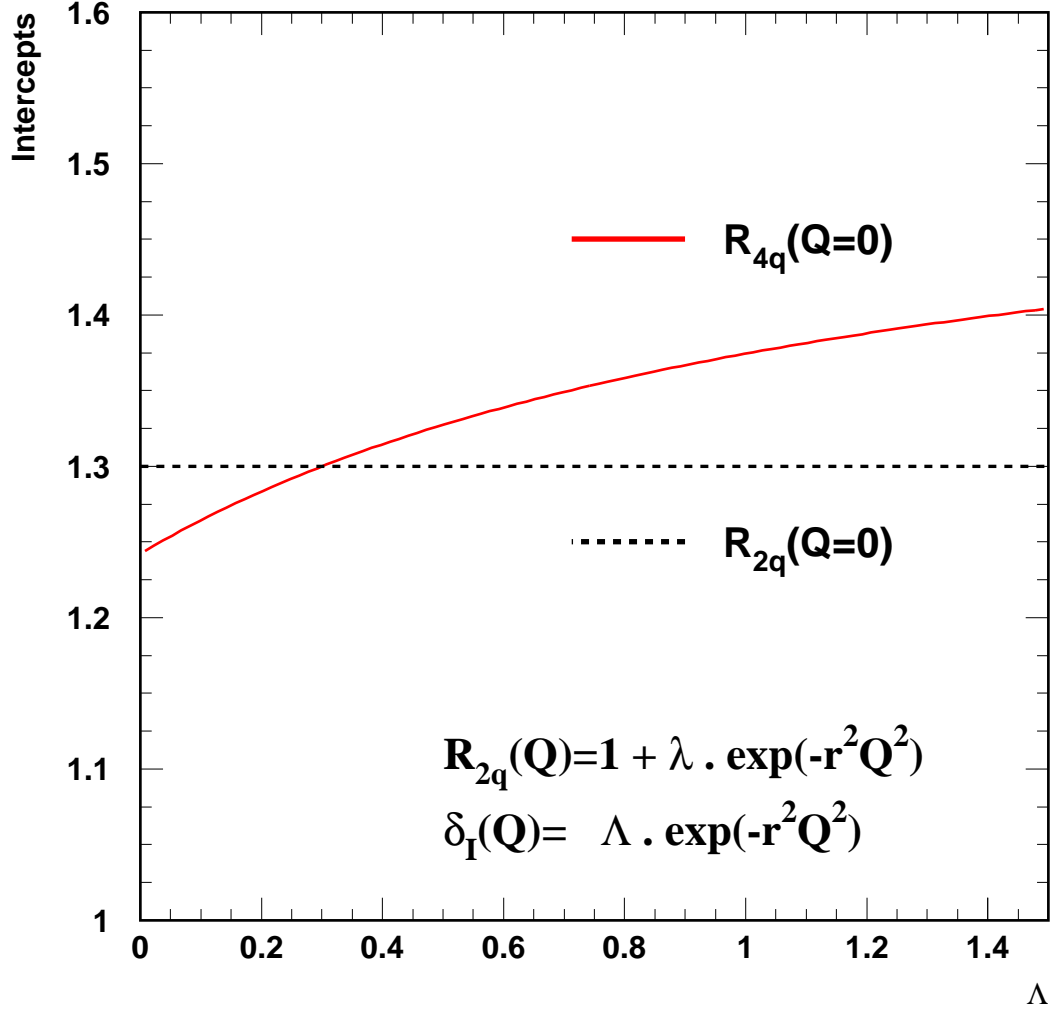
Figure 4: DELPHI: The value of the normalized density $R_{4q}(Q) \equiv R_2^{\mathrm{WW}}(Q)$, Eq.(77) in the text, and of $R_{2q}(Q) \equiv R_2^{\mathrm{W}}(Q)$, at $Q = 0$, as a function of the parameter Λ in $\delta_I(Q) = \Lambda \exp\left(-r^2 Q^2\right)$.
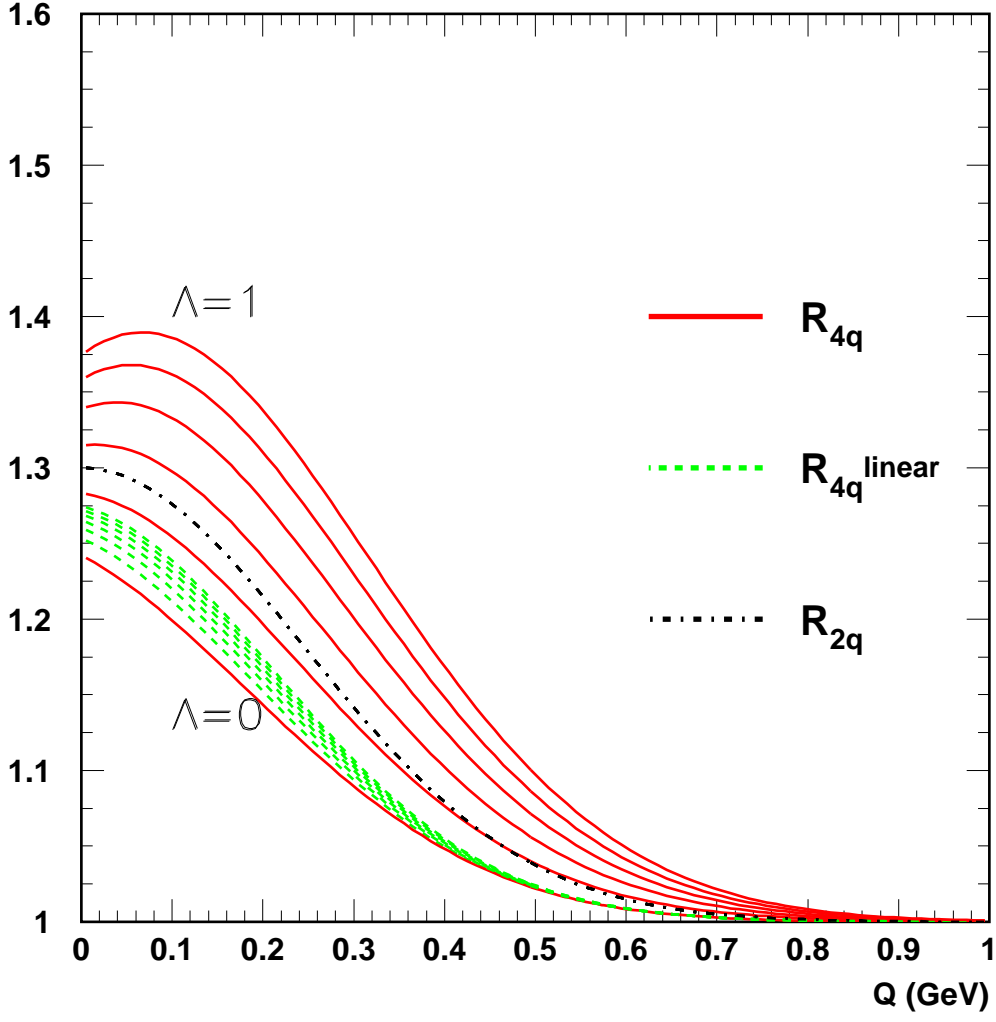
Figure 5: DELPHI: The normalized density $R_{4q}(Q) \equiv R_2^{\mathrm{WW}}(Q)$, Eq.(77) in the text, using as input the DELPHI parameterization of $R_{2q}(Q) \equiv R_2^{\mathrm{W}}(Q)$ and with $\delta_I(Q) = \Lambda \exp\left(-r^2 Q^2\right)$; Λ varies in the range $0.0 - 1.0$ in steps of 0.2. Also shown is the distribution corresponding to the so-called Linear Scenario in ref. [16].